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EFFECT OF PARTICLE SHAPE ON SOME BULK SOLIDS PROPERTIES.

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ABSTRACT

Apparent densities have been measured of glass beads of various shapes, to study the effect of the shape factor on bed porosity. It was found that the porosity was *linearly related to the shape factor*. The densities were carried out in graduates of different diameters to study the wall effect. It was found that the porosity (ε) is a function of graduate diameter (D) by a function of the type: $\ln[\varepsilon\text{-}Q] = -kD + q$, where Q is the smallest porosity under the pouring and shaking conditions.

MATERIALS AND METHODS

Glass beads and rods were obtained from the Proper Glass Co and the Flexolite Mfg. Co. in St. Louis. Three types were obtained: (a)

Table I. Dimensions of Glass Beads Used.

	Sphere	Rod A		Rod B	
	D*	D	h*	D	h
No. measured	10	10		10	
Size (cm)¢	0.525	0.510	0.585	0.415	0.916
Diameter (cm) d_V	0.525	0.611		0.714	
Shape Factor	4.835	5.547		5.728	
Volume/100 (cm^3)	7.57	11.94		19.08	
Weight/100 (gram)	20.196¶	25.751¶		53.087¶	

* D denotes diameter of either sphere or base of the cylinder and *h* denotes height of the cylinder. ¢Average of 6 determinations. ¶Average of six determinations.

spherical, (b) isometric (cylindrical) rods (denoted Rod A in the following) and (c) cylindrical rods with a height twice the diameter.

The dimensions of the beads were checked by a Starrett micrometer #1D102.

The volume diameter (d_V) is the diameter of the sphere which has the same volume as the cylindrical rod[1], i.e.

$$\pi d_V^3 = \pi h d_c^2/4 \quad \text{(Eq. 1)}$$

or

$$(d_V) = [1.5\ hd_C^2]^{2/3} \quad \text{(Eq. 2)}$$

where d_C is the diameter of the base of the cylindrical rod.

The shape factor[2,3] is denoted Γ and is given by

$$\text{Area} = \Gamma\ (\text{Volume})^{2/3} \quad \text{(Eq. 3)}$$

i.e. for a sphere:

$$\Gamma\ (\pi d^3/6)^{2/3} = \pi d^2 \ \text{ or } \Gamma = 6^{2/3}\pi^{1/3} = 4.835 \quad \text{(Eq. 4)}$$

and for a cylinder

$$\Gamma\ [h\pi d_C^2/4] = [2\pi d_C^2/4] + \pi d_C h \quad \text{(Eq. 5)}$$

This shows that a cylinder is only isometric when its height equals its diameter (since this is the only situation where Γ becomes independent of d_C and h). The shape factors in Table I have been calculated according to Eq.s 4 and 5.

Apparent densities of the glass beads were determined by pouring a given number of grams (G) into graduates of volume v cm^3. The amounts and the volumes are shown in Table II, as is the inside diameter (d_g cm) of the graduates. The cascaded volume was then recorded. The graduate was then subjected to 30 seconds of vibration on a 60 cycle syntron with a flow feeder base (model FO, 7 amps).

Table 2 Experimental Parameters

Graduate Volume (v cm^3)	Grams (G) of Beads	Inside Diameter d_c (cm)
250	100	3.69
100	100	2.57
50	50	2.02
25	25	1.89
25	25	1.55
10	10	1.28

The number of grams were weighed precisely, and is close to the number shown in the table. The weight of 100 beads was then determined so that the weight of beads could be converted to number of beads. Knowing the volume of the beads from their dimensions then allows calculation of the true solids volume (V_s). The apparent volume (V_a) is obtained directly by measurement of the volume of the beads in the graduate. The porosity (ε) was then calculated as:

$$\varepsilon = 1 - (V_s/V_a) \qquad \text{(Eq. 6)}$$

RESULTS AND DISCUSSION

The porosities (ε) are plotted as a function of graduate diameter in Fig. 1. (The data for Rods B are left out for graphical clarity,

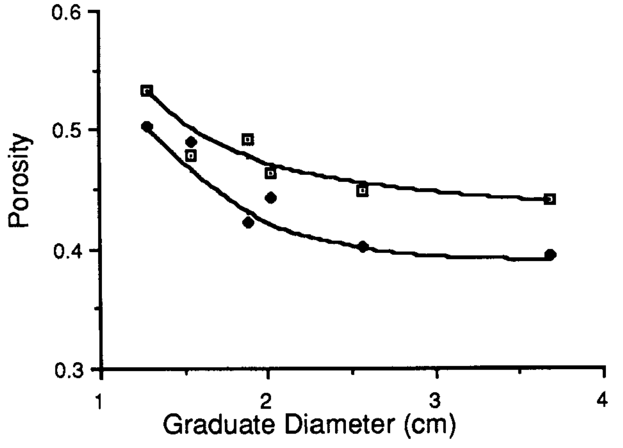


Fig. 1. Bed porosities as a function of graduate diameter for spheres (square open symbol) and Rod A (closed diamond symbol). The data for Rod B have been left out for graphical clarity.

but are shown in the subsequent Fig. 2). It is apparent[4] that the porosity decreases as the graduate diameter increases.

Actual unambiguous porosity (denoted true porosity, Q, in the following) can only be defined in an infinite bed (so that the wall effects are completely absent). The value of Q for close packing of spheres (tetrahedral) would be 0.26 (see Appendix). For cylinders, the value of Q may be visualized as the packing where all the cylinders are stacked, one on top of the other. This is depicted in Fig. 2. It is seen that the value of Q can be deduced from the triangular cross-section cylinder, ABC which contains one half circular cross-section cylinder, since the area of each of the circle segments is 1/6 of the area of the whole circle. The cross-sectional area of the triangular cylinder ABC is known, since the height of the triangle is $d_c \cdot [\sqrt{3}/2]$, i.e. per unit length of the two types of

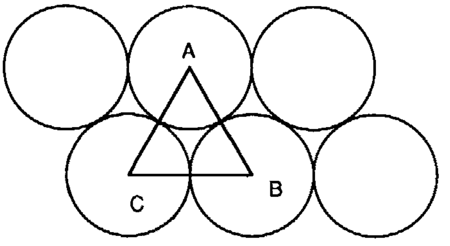
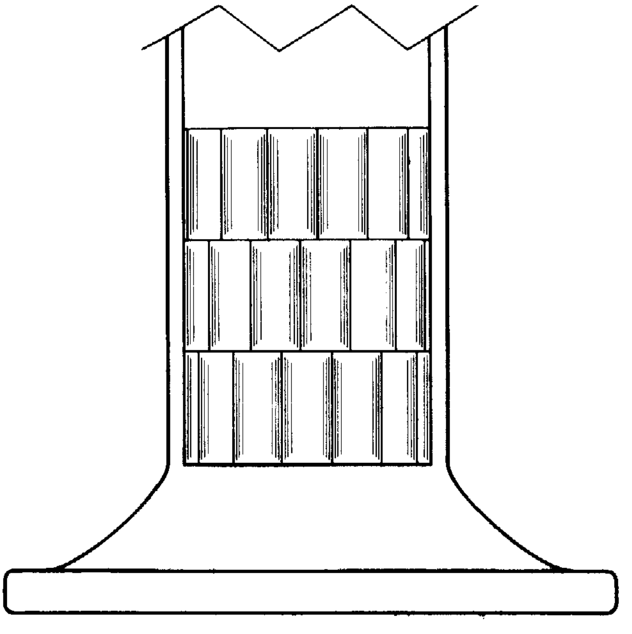
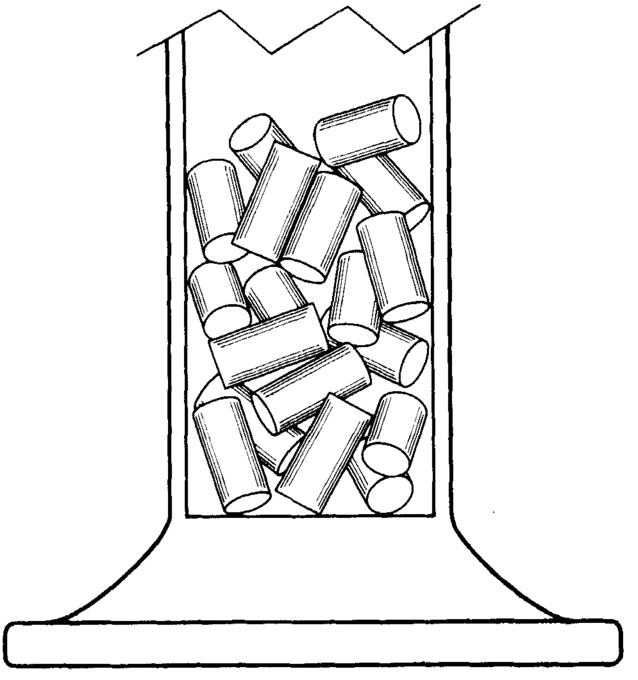


Fig. 2A. Schematic of close-packed bed of cylinders seen from the side. The unit cell is the triangular cross-section cylinder ABC containing 1/2 circular cross-section cylinder



SIDE VIEW
RODS B ON END

Fig. 2B. Side view of Rods B on End.



SIDE VIEW
RODS B RANDOM DISTRIBUTION

Fig. 2C. Side view of rods B in random distribution

cylinder one may now calculate Q:

$$Q_{cylinder} = 1 - \{0.5 \cdot (\pi \cdot d_c^2/4\}/\{d_c(d_c \cdot [\sqrt{3}/2])\} = 0.1$$

so that the porosity at close packing is less than that for for a bed of packed spheres. It is obvious, however, that it would be much more difficult (actually impossible) for a bed of rods to attain its minimum porosity.

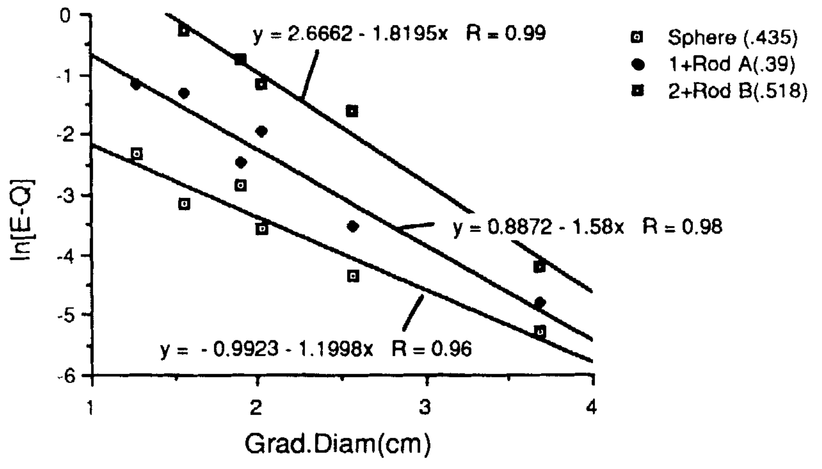


Fig. 3. Data from Fig. 1 plotted according to Eq. 7.

If the data are plotted as

$$\ln[\varepsilon\text{-}Q] = \mu - \beta \cdot d_b \quad \text{(Eq. 7)}$$

then as seen in Fig. 3, good straight lines result. μ and β are here constants, μ being dimensionless and β having units of reciprocal length. It is noted that the values of Q, however, fall short of the values for close packing.

It has been pointed out by Ridgway and Rupp (6) that the apparent densities should be a function of shape factors, and as seen from Fig. 3 they indeed are.

The consolidation ratio[5], P, also is a function of both shape and container. Fig. 4 shows the ratio of cascaded to vibrated density for spheres and for isometric rods (Rod A). For Rod B, the consolidation data were more scattered. It would appear that the consolidation ratio

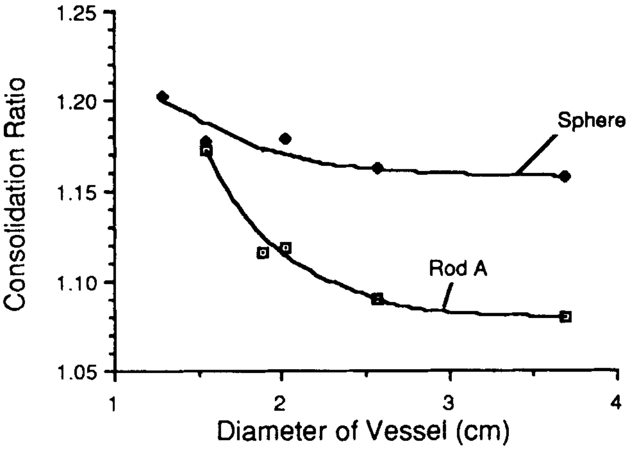


Fig. 4. Consolidation ratios, P, for spheres and for Rod A, as a function of diameter of the cylinder in which the apparent density was determined.

decreases with increasing cylinder diameter. The data, again, can be fitted to an equation of the type:

$$\ln[P-P^*] = \text{ø}^* = \beta^* x \qquad \text{(Eq. 8)}$$

The data point out the necessity for specifying the graduate diameter each time an apparent density is reported. It also points to the fact that close packing is never really achieved.

This has some interesting consequences for instance in compression. It is known that there is a wall effect as far as the radial packing density is concerned[7]. Another aspect of bed densities is, however, also of great importance: It is often assumed that the first stage of compression of a powder in a tablet die is the change of the

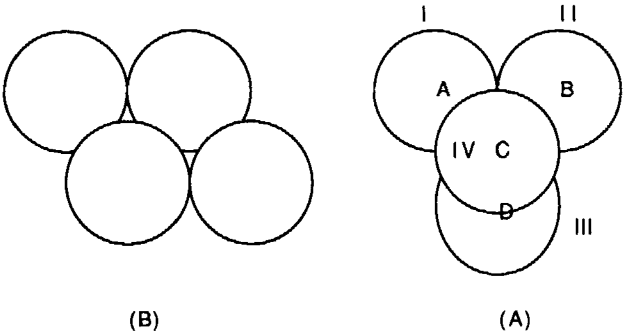


Fig. 5. (A) Spheres in close packing. The "top" sphere lies in the crevice of the three supporting spheres. It is noted that their centers form a tetrahedron with side h. All angles are 60°. (B) Side view of layers of the spheres. It is seen that the unit cell is a parallelipiped (shown in Fig. 6).

powder bed in the die cavity from its cascaded density to its close packed density. From the above data it would seem that the latter would be highly unlikely, and that the actual point where bonds will start to form in the die will be at apparent densities which are less than the close packed density.

APPENDIX

Consider the spheres in close packing shown in Fig. 5. As seen from Fig. 5B and Fig. 6, it is necessary to know the height, h', of the parallelipiped in order to calculate the theoretical porosity. Using the nomenclature of Fig. 5A, Fig. 6 is constructed. It is seen in Fig. 6C that the triangle BED has sides BD = d and BE = ED = $(\sqrt{3}/2)\cdot d$. The distance which is sought is BF, and it is seen from Fig. 6C that (x being distance EF) from triangle EFB

$$(BF)^2 = (3/4)d^2 - x^2 \qquad \text{(Eq. 9)}$$

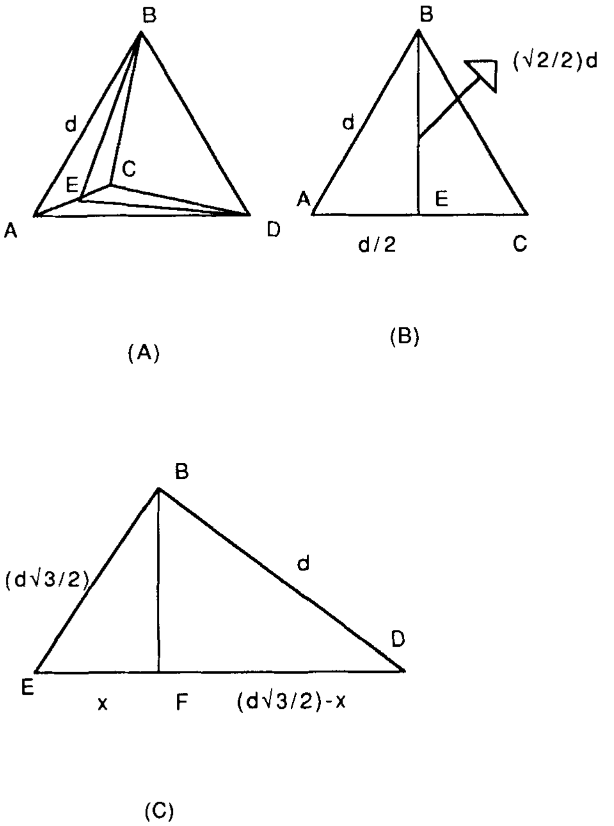


Fig. 6 (A) Tetrahedron formed by centers of spheres in Fig. 5A. (B). The side of the tetrahedron, with nomenclature consisten with Fig. 6A. (C). Normal cut along lines shown in Fig. 6A.

and from triangle BFD

$$(BF)^2 = d^2 - [(\sqrt{3}/2)\cdot d - x]^2 \qquad \text{(Eq. 10)}$$

Equating Eqs 9 and 10 then gives:

$$x = d/(2\sqrt{3}) \qquad \text{(Eq. 11)}$$

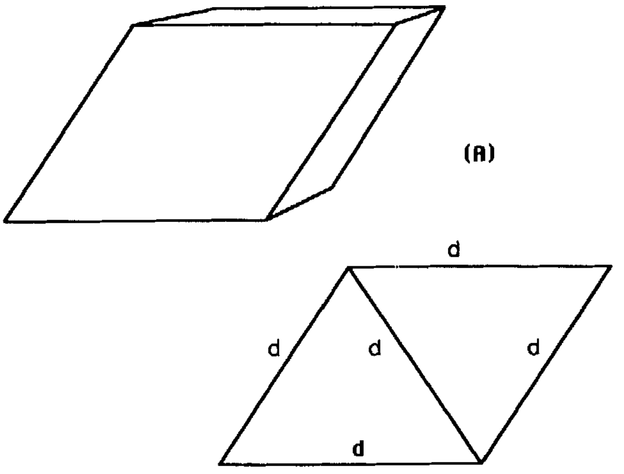


Fig. 7. (A) Unit cell (parallelipiped). (B) Base of the parallelipiped, a trapezoid, essentially formed by two equilateral triangles.

which inserted in Eq. 9 gives

$$BF = (\sqrt{3}.2)d^2 \quad \text{(Eq. 12)}$$

The unit cell is a parallelipipid, with a base as shown in Fig. 7, i.e. composed of a trapezoid of base angle 60. Its area is $(\sqrt{3}.2)d^2$, so that the volume of the parallelipipid is

$$(\sqrt{3}.2)d^2 \cdot [(\sqrt{3}.2)d^2] = (\sqrt{2}.2)d^3 \quad \text{(Eq. 13)}$$

Each corner of the unit cell is occupied by 1/8 sphere, and there are eight corners, so that the unit cell contains one sphere. The solids fraction, hence, is

$$\sigma = [\pi/6]d^3/\{(\sqrt{2}/2)d^3\} = 0.74 \quad \text{(Eq. 14)}$$

i.e. the porosity is 0.26.